FORECASTING STOCK MARKET VOLATILITY: EVIDENCE FROM FOURTEEN COUNTRIES

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Abstract

This paper evaluates the out-of-sample forecasting accuracy of seven models for weekly volatility in fourteen stock markets. Volatility is defined as within-week standard deviation of continuously compounded daily returns on the stock market index of each country for the period December 1987 to December 1997. Total volatility series include 522 weeks. The first half of the sample (261 weeks) is retained for the estimation of parameters while the second half is for the forecast period. The following models are employed: a random walk model, a historical mean model, moving average models, weighted moving average models, exponentially weighted moving average models, an exponential smoothing model, and a regression model. We first use the standard loss functions to evaluate the performance of the competing models: the mean error, the mean absolute error, the root mean squared error, and the mean absolute percentage error. We also employ the asymmetric loss functions to penalise under/over-prediction.

JEL Classification: C22; C53; G12; G1

I. INTRODUCTION

The financial economic research has no consensus on the relative quality of volatility forecasts in the financial markets. Different studies recommend different forecasting methods. For example, in forecasting volatility of stock exchange markets, Tse (1991), and Tse and Tung (1992) favour exponential weighted moving average model in Japan and Singapore. However, Dimson and Marsh (1990) analysing the U.S. equity market show superiority of simple regression and exponential smoothing models.

Brailsford and Faff (1996) investigate the out-of-sample predictive ability of several models of stock market volatility in Australia. In the measurement of the performance of the models, in addition to symmetric loss functions, they use asymmetric loss functions to penalise under/over prediction. They conclude that the ARCH class of models and a simple regression model provide superior forecast of the volatility. However, the various model ranking are shown to be sensitive to the error statistics used to assess the accuracy of the forecasts.

On the other hand, The evidence with respect to foreign exchange markets is presented by West and Cho (1995), Brooks and Burke (1998), and Balaban (1999). West and Cho (1995) can not show superiority of any forecasting models.

In the finance literature, the existing evidence about the relative quality of volatility forecasts is related to an individual country's stock market: the USA (Akgiray, 1989), the UK (Dimson and Marsh, 1990), Japan (Tse, 1991), Singapore (Tse and Tung, 1992), Australia (Brailsford and Faff, 1996), Switzerland (Adjaoute, Bruand and Gibson-Asner, 1998), Turkey (Balaban, 1998). Moreover, most of the previous researches focus on the forecasting monthly stock market volatility. The present study is primarily based on Brailsford and Faff (1996), and provides an international evidence from fourteen countries with respect to weekly stock market volatility.

The rest of the paper is organised as follows: In the second section, data and methodology are described, in the third section empirical results are presented, and finally in the fourth section the paper is concluded.

II. DATA AND METHODOLOGY

We employ daily observations of stock market indices of fourteen countries for the period December 1987 to December 1997. The data have been obtained from *Datastream*, and the investigated countries (indices) are Belgium (Brussels All Shares Price Index), Canada (Toronto SE 300 Composite Price Index) , Denmark (Copenhagen SE General Price Index), Finland (Hex General Price Index), Germany (Faz General Price Index), Hong Kong (Hang Seng Price Index), Italy (Milan Comit General Price Index), Japan (Nikkei 500 Price Index), the Netherlands (CBS All Share General Price Index), the Philippines (Philippines SE Composite Price Index), Singapore (Singapore All Share Price Index), Thailand (Bangkok S.E.T. Price Index), the UK (FTSE All Share Index) and the USA (NYSE Composite Index). The continuously compounded weekly returns are calculated as follows:

$$R_{w,t} = \ln\left(I_{w,t} / I_{w,t-1}\right) \tag{1}$$

$$\sigma_{a,w} = \left[\frac{1}{(n-1)} \sum_{t=1}^{n} \left(R_{w,t} - \mu_w\right)^2 \right]^{0.5} \tag{2}$$

where $I_{w,t}$ and $R_{w,t}$ denote the value of stock market index and continuously compounded return on trading day t in week w, respectively. We define weekly realised volatility as the within-week standard deviation of continuously compounded weekly returns as follows:

$$\mu_w = (1/n) \sum_{t=1}^{n} R_{w,t} \tag{3}$$

Mean daily index return and within-week standard deviation of daily returns in week w are respectively shown by μ_w and $\sigma_{a,w}$. The number of trading days in a week is given by n. In the data set for each country there are 522 weekly volatility observations. In the analysis, first 261 of the observations from December 1987 to November 1992 are used for estimation, and the second 261 observations from December 1992 to December 1997 are used for forecasting. In the Table 1 summary statistics for within-week standard deviations of returns in whole period, estimation period, and forecast period are presented. The table shows that in four countries, Canada, Finland, Hong Kong and Italy, standard deviations in forecast period are higher than estimation period. In most of the countries, in the forecast period, standard deviations decline.

Table 1. SUMMARY STATISTICS: Within-Week Standard Deviations

	Whole Period				Estimation Period				Forecast Period			
	Mean	Standard Dev.	Skewness	Kurtosis	Mean	Standard Dev.	Skewness	Kurtosis	Mean	Standard Dev.	Skewness	Kurtosis
BEL	0.0051	0.0039	2.6991	13.4987	0.0054	0.0047	2.3840	10.2340	0.0048	0.0029	2.6700	15.9900
CAN	0.0048	0.0030	3.2658	25.1900	0.0048	0.0026	2.2490	12.4650	0.0049	0.0033	3.6600	28.3600
DEN	0.0050	0.0035	3.8511	30.5222	0.0051	0.0037	2.9370	17.2990	0.0048	0.0034	5.0000	49.0000
FIN	0.0084	0.0059	2.6860	18.9146	0.0065	0.0052	2.3570	10.9550	0.0103	0.0060	3.4800	27.0300
GER	0.0086	0.0062	4.9616	45.4618	0.0095	0.0073	5.0010	41.0710	0.0078	0.0047	3.1600	22.0200
HON	0.0118	0.0107	5.2704	45.1534	0.0105	0.0104	6.1310	55.3760	0.0132	0.0109	4.7000	38.9700
ITAL	0.0097	0.0057	2.3184	14.1141	0.0087	0.0059	2.3500	11.2360	0.0107	0.0053	2.6400	20.2300
JAP	0.0089	0.0067	2.4076	11.0324	0.0094	0.0080	2.1890	9.0240	0.0083	0.0052	2.2100	9.9600
NET	0.0071	0.0043	2.4407	12.3680	0.0071	0.0044	2.4080	11.5750	0.0070	0.0043	2.4700	13.1400
PHI	0.0126	0.0076	1.3473	5.3678	0.0142	0.0081	1.1380	4.6120	0.0110	0.0067	1.5900	6.6500
SNG	0.0074	0.0056	3.1548	17.6716	0.0074	0.0062	3.2390	17.8920	0.0073	0.0051	2.8500	14.9000
THA	0.0128	0.0091	2.0638	8.5456	0.0129	0.0103	2.1110	8.2760	0.0127	0.0078	1.7400	6.8700
UK	0.0063	0.0032	2.2687	11.9167	0.0071	0.0036	2.2840	10.5610	0.0055	0.0024	1.6300	9.7100
US	0.0064	0.0039	2.9443	20.3649	0.0073	0.0041	2.4200	13.8160	0.0056	0.0036	4.0100	35.5700

Whole period includes the whole sample (522 weeks).
Estimation period covers the first 216 observations.
Forecast period covers the second 216 weeks.

The following models are employed as forecast competitors:

a) Random walk model:

This model says that the best forecast of this week's volatility is the last week's realised volatility.

$$\sigma_{f,w}(RW) = \sigma_{f,w-1} \tag{4}$$

where w = 262, ..., 522.

b) Historical mean model:

According to this model, the best forecast of this week is average of all past observations available.

$$\sigma_{f,w}(HM) = \frac{1}{w-1}\sum_{j=1}^{w-1}\sigma_{a,j} \tag{5}$$

where w = 262, ..., 522.

c) Moving average (MA-α) model:

This model says that the best forecast of this week's volatility is equally weighted average of realised volatilities in the last α weeks.

$$\sigma_{f,w}(MA(\alpha)) = \frac{1}{\alpha}\sum_{j=1}^{\alpha}\sigma_{a,w-j} \tag{6}$$

where w = 262, ..., 522, and α = 4, 6, 12, 24, 36, 52. The (arbitrarily) chosen values of α represent different horizons from the very short to the long terms.

d) Weighted moving average (WMA-α) model:

In the WMA-α model, weight of each observation is not equal like in MA-α model (Liljeblom and Stenius (1997))

$$\sigma_{f,w}(WMA(\alpha)) = \sum_{j=1}^{\alpha}\lambda_i\sigma_{a,w-j} \tag{7}$$

where w = 262, ..., 522, and α = 4, 6, 12, 24, 36, 52.

In the equation (7), the weight of each observation, λ_i, declines by 10%, giving the highest (lowest) weight to the newest (oldest) information.

e) Exponentially smoothing (ES) model:

In the ES model, the forecast of volatility is a function of the immediate past forecast and the immediate past observed volatility (Dimson and Marsh (1990); Brailsford and Faff (1996)).

$$\sigma_{f,w}(ES) = \theta\sigma_{f,w-1}(ES) + (1-\theta)\sigma_{a,w-1} \tag{8}$$

where w = 262, ..., 522.

The smoothing parameter (θ) is restricted to lie between zero and one. Following the previous research, we determine the optimal value of θ empirically. To this end, we start an initial value of θ, zero in our case, and increase it by 0.01 each time until we obtain unity for q ((Brailsford and Faff (1996)).

f) Exponentially weighted moving average (EWMA-α) model:

In this model, the past observed volatility is replaced by the α-week moving average forecast; ie., the forecast of the MA-α model (Tse, 1991; Tse and Tung, 1992; and Brailsford and Faff, 1996).

$$\sigma_{f,w}(EWMA-\alpha) = \lambda\sigma_{f,w-1}(EWM-\alpha) + (1-\lambda)\sigma_{a,w}(MA-\alpha) \tag{9}$$

where w = 262, ..., 522, and α = 4, 6, 12, 24, 36, 52. Similar to the MA-α models, the (arbitrarily) chosen values represent different horizons from the very short to the long terms.

g) Regression (REG) model:

We first run the regression below for the sample where w is between one and 261.

$$\sigma_{a,w} = c + \beta\sigma_{a,w-1} + u_{w-1} \tag{10}$$

Then we construct the forecast for the first week of the forecast period (w = 262) using the estimated regression parameters:

$$\sigma_{f,w}(REG) = c + \beta\sigma_{a,w-1} \tag{11}$$

We update the regression equation weekly; ie., each week we drop the oldest observation and add the last or newest observation. Thus we use the same number of observations in each case. Hence for each country the total estimation procedure requires estimation of 261 regressions to obtain out-of-sample forecasts of weekly volatility. Note that this procedure lets us depend on the time-varying parameters for each forecast.

III. FORECAST EVALUATION AND EMPIRICAL RESULTS

We compare the forecast performance of each model through both symmetric and asymmetric error statistics.

a) Symmetric Error Statistics

Four commonly used loss functions or error statistics: the mean error (ME), the mean absolute error (MAE), the root mean squared error (RMSE), and the mean absolute percentage error (MAPE) are employed to measure the performance of the forecasting models.

$$ME = \frac{1}{261}\sum_{m=262}^{522}\left(\sigma_{f,w} - \sigma_{a,w}\right) \tag{12}$$

$$MAE = \frac{1}{261}\sum_{m=262}^{522}\left|\sigma_{f,w} - \sigma_{a,w}\right| \tag{13}$$

$$RMSE = \left[\frac{1}{261}\sum_{m=262}^{522}\left(\sigma_{f,w} - \sigma_{a,w}\right)^2\right]^{0.5} \tag{14}$$

$$MAPE = \frac{1}{261}\sum_{m=262}^{522}\left|\frac{\sigma_{f,w} - \sigma_{a,w}}{\sigma_{a,w}}\right| \tag{15}$$

$\sigma_{f,w}$ and $\sigma_{a,w}$ denote the volatility forecast and the realised volatility in week w , respectively.

Table 2, Table 3, Table 4, and Table 5 provide results of forecast error statistics for each model according to symmetric error measures, (ME, MAE, RMSE, and MAPE respectively).

Table 2 illustrates that according to ME criteria, EWMA and ES model under-predict the volatility in all of the stock markets. Moreover, except for HM and REG, all models under-predict the volatility in most of the stock markets.

Table 2:

	RW	HM	MA-12	WMA-12	EWMA-12	ES	REG
BEL	-0.009	0.241	-0.120	-0.101	-0.236	-0.240	0.152
CAN	-0.034	-0.228	-0.160	-0.132	-0.211	-0.210	-0.219
DEN	0.025	0.243	-0.006	-0.044	-0.096	-0.010	0.230
FIN	0.044	-2.700	-0.005	-0.032	-0.131	-0.130	-1.776
GER	-0.041	1.023	-0.210	-0.177	-0.321	-0.320	0.785
HON	-0.032	-1.814	-0.600	-0.479	-0.020	-0.002	-1.192
ITAL	0.014	-1.257	0.001	0.012	-0.246	-0.250	-0.905
JAP	-0.089	0.671	-0.270	-0.238	-0.274	-0.270	0.257
NET	-0.042	-0.326	-0.270	-0.222	-0.309	-0.310	-0.215
PHI	-0.028	2.142	-0.310	-0.249	-0.347	-0.350	1.419
SNG	-0.044	-0.064	-0.230	-0.182	-0.277	-0.280	-0.043
THA	-0.017	-0.188	-0.130	-0.095	-0.301	-0.300	-0.105
UK	-0.006	1.137	-0.005	-0.048	-0.133	-0.130	0.791
US	-0.019	0.945	-0.200	-0.165	-0.310	-0.310	0.675

Results of MAE in Table 3 indicate that in ranking, HW gives the second worst results in the forecasting, after RW model. However, ES model gives the most accurate forecasts. In 12 countries, ES model is the best model. In the other countries, Canada and Hong Kong, REG and WMA-12 are the best models respectively. In general ranking, WMA-12 provides the second best forecast. MA-12, EWMA-12 and REG models rank third, fourth and fifth respectively.

As shown in Table 4, according to RMSE criteria, ES model is the best performing model. On the other hand, RW model provides the worst forecasts in 12 countries. For the other countries, Hong Kong and the Netherlands, HM is the worst model. In ranking, WMA-12 and MA-12 are again the second and the third best models respectively.

Finally, Table 5 shows the results according to MAPE criteria, like under previous criterias, ES model gives relatively more accurate results than the other models. However, according to MAPE, even though in general HM is the second least performing model, it gives the best results in two countries, Finland and Italy.

In summary, ES model is consistently the best performing model according to MAE, RMSE, and MAPE. On the other hand, HM is the least performing model when MAE, and RMSE are used, and it is the second least performing one when MAPE is used.

In all of the countries, except in Finland, Italy, the Netherlands, Singapore, and the US, symmetric error statistics provide quite consistent results for the performance of the models.

Table 3:

		RW	HM	MA-12	WMA-12	EWMA-12	ES	REG
BEL	Actual	2.363	2.076	1.854	1.837	1.860	1.809	1.946
	Relative	1.000	0.879	0.785	0.778	0.787	0.765	0.824
	Rank	7	6	3	2	4	1	5
CAN	Actual	2.477	2.070	2.069	2.023	2.043	1.954	1.950
	Relative	1.000	0.836	0.835	0.817	0.825	0.789	0.787
	Rank	7	6	5	3	4	2	1
DEN	Actual	2.477	2.126	2.019	1.991	2.030	1.949	2.016
	Relative	1.000	0.858	0.815	0.804	0.820	0.787	0.814
	Rank	7	6	4	2	5	1	3
FIN	Actual	5.234	4.146	3.879	3.891	3.886	3.808	3.884
	Relative	1.000	0.792	0.741	0.743	0.742	0.727	0.742
	Rank	7	6	2	5	4	1	3
GER	Actual	3.463	3.512	2.768	2.714	2.735	2.618	3.132
	Relative	0.986	1.000	0.788	0.773	0.779	0.745	0.892
	Rank	6	7	4	2	3	1	5
HON	Actual	6.406	6.119	5.286	5.232	5.351	5.614	5.518
	Relative	1.000	0.955	0.825	0.817	0.835	0.876	0.861
	Rank	7	6	2	1	3	5	4
ITA	Actual	4.815	3.741	3.686	3.749	3.624	3.602	3.724
	Relative	1.000	0.777	0.765	0.779	0.753	0.748	0.774
	Rank	7	5	3	6	2	1	4
JAP	Actual	4.055	3.811	3.424	3.335	3.455	3.259	3.405
	Relative	1.000	0.940	0.844	0.823	0.852	0.804	0.840
	Rank	7	6	4	2	5	1	3
NET	Actual	2.691	2.901	2.439	2.382	2.378	2.327	2.408
	Relative	0.927	1.000	0.841	0.821	0.820	0.802	0.830
	Rank	6	7	5	3	2	1	4
PHI	Actual	5.292	5.775	4.433	4.344	4.454	4.300	4.943
	Relative	0.916	1.000	0.768	0.752	0.771	0.745	0.856
	Rank	6	7	3	2	4	1	5
SNG	Actual	3.844	3.314	3.160	3.136	3.185	3.125	3.144
	Relative	1.000	0.862	0.822	0.816	0.829	0.813	0.818
	Rank	7	6	4	2	5	1	3
THA	Actual	6.123	5.738	5.246	5.176	5.298	5.054	5.365
	Relative	1.000	0.937	0.857	0.845	0.865	0.825	0.876
	Rank	7	6	3	2	4	1	5
UK	Actual	2.114	2.207	1.728	1.687	1.742	1.659	1.932
	Relative	0.958	1.000	0.783	0.764	0.789	0.752	0.875
	Rank	6	7	3	2	4	1	5
US	Actual	2.604	2.776	2.025	1.989	2.017	1.942	1.983
	Relative	0.938	1.000	0.730	0.717	0.726	0.699	0.714
	Rank	6	7	5	3	4	1	2

Table 4. RMSE

		RW	HM	MA-12	WMA-12	EWMA-12	ES	REG
BEL	Actual	3.334	2.958	2.659	2.658	2.688	2.647	2.762
	Relative	1.000	0.887	0.798	0.797	0.806	0.794	0.828
	Rank	7	6	3	2	4	1	5
CAN	Actual	3.892	3.267	3.235	3.187	3.220	3.130	3.140
	Relative	1.000	0.839	0.831	0.819	0.827	0.804	0.807
	Rank	7	6	5	3	4	1	2
DEN	Actual	3.899	3.379	3.256	3.239	3.278	3.211	3.230
	Relative	1.000	0.867	0.835	0.831	0.841	0.824	0.828
	Rank	7	6	4	3	5	1	2
FIN	Actual	7.43	6.556	5.892	5.879	5.902	5.835	6.117
	Relative	1.000	0.882	0.793	0.791	0.794	0.785	0.823
	Rank	7	6	3	2	4	1	5
GER	Actual	5.101	4.836	4.242	4.198	4.272	4.154	4.462
	Relative	1.000	0.948	0.832	0.823	0.837	0.814	0.875
	Rank	7	6	3	2	4	1	5
HON	Actual	10.797	11.018	9.697	9.591	9.719	9.664	9.708
	Relative	0.980	1.000	0.880	0.870	0.882	0.877	0.881
	Rank	6	7	3	1	5	2	4
ITA	Actual	6.750	5.476	5.153	5.166	5.176	5.109	5.329
	Relative	1.000	0.811	0.763	0.765	0.767	0.757	0.789
	Rank	7	6	2	3	4	1	5
JAP	Actual	5.634	5.222	4.874	4.763	4.915	4.686	4.818
	Relative	1.000	0.927	0.865	0.845	0.872	0.832	0.855
	Rank	7	6	4	2	5	1	3
NET	Actual	3.776	4.324	3.452	3.387	3.472	3.329	3.520
	Relative	0.873	1.000	0.798	0.783	0.803	0.770	0.814
	Rank	6	7	3	2	4	1	5
PHI	Actual	7.198	7.083	6.082	6.001	6.129	6.007	6.282
	Relative	1.000	0.984	0.845	0.834	0.851	0.835	0.873
	Rank	7	6	3	1	4	2	5
SNG	Actual	5.611	5.068	4.730	4.734	4.819	4.756	4.720
	Relative	1.000	0.903	0.843	0.844	0.859	0.848	0.841
	Rank	7	6	2	3	5	4	1
THA	Actual	8.447	7.843	7.251	7.134	7.333	7.065	7.164
	Relative	1.000	0.928	0.858	0.845	0.868	0.836	0.848
	Rank	7	6	4	2	5	1	3
UK	Actual	2.752	2.720	2.320	2.278	2.329	2.251	2.435
	Relative	1.000	0.988	0.843	0.828	0.846	0.818	0.885
	Rank	7	6	3	2	4	1	5
US	Actual	3.940	3.774	3.211	3.192	3.203	3.168	2.513
	Relative	1.000	0.958	0.815	0.810	0.813	0.804	0.638
	Rank	7	6	5	3	4	2	1

The root mean squared error, (RMSE), actual figures must be multiplied by 10^{-2}.

Table 5. MAPE

		RW	HM	MA-12	WMA-12	EWMA-12	ES	REG
BEL	Actual	0.561	0.593	0.476	0.474	0.470	0.459	0.538
	Relative	0.945	1.000	0.802	0.798	0.792	0.774	0.907
	Rank	6	7	4	3	2	1	5
CAN	Actual	0.577	0.528	0.522	0.515	0.499	0.489	0.495
	Relative	1.000	0.915	0.904	0.893	0.866	0.849	0.858
	Rank	7	6	5	4	3	1	2
DEN	Actual	0.625	0.651	0.543	0.535	0.547	0.530	0.616
	Relative	0.960	1.000	0.834	0.822	0.841	0.814	0.947
	Rank	6	7	3	2	4	1	5
FIN	Actual	0.691	0.422	0.553	0.553	0.553	0.535	0.461
	Relative	1.000	0.610	0.800	0.800	0.800	0.775	0.668
	Rank	7	1	5	6	4	3	2
GER	Actual	0.569	0.667	0.471	0.465	0.455	0.443	0.600
	Relative	0.852	1.000	0.707	0.697	0.682	0.664	0.899
	Rank	5	7	4	3	2	1	6
HON	Actual	0.549	0.524	0.474	0.468	0.482	0.491	0.484
	Relative	1.000	0.955	0.864	0.853	0.878	0.895	0.882
	Rank	7	6	2	1	3	5	4
ITA	Actual	0.557	0.427	0.442	0.447	0.438	0.431	0.431
	Relative	1.000	0.766	0.792	0.802	0.786	0.774	0.773
	Rank	7	1	5	6	4	3	2
JAP	Actual	0.609	0.662	0.507	0.501	0.511	0.491	0.563
	Relative	0.921	1.000	0.766	0.757	0.772	0.742	0.851
	Rank	6	7	3	2	4	1	5
NET	Actual	0.460	0.519	0.406	0.400	0.385	0.387	0.442
	Relative	0.886	1.000	0.782	0.770	0.742	0.745	0.850
	Rank	6	7	4	3	1	2	5
PHI	Actual	0.573	0.839	0.510	0.503	0.504	0.490	0.681
	Relative	0.683	1.000	0.608	0.599	0.601	0.584	0.812
	Rank	5	7	4	2	3	1	6
SNG	Actual	0.608	0.586	0.533	0.529	0.532	0.522	0.551
	Relative	1.000	0.964	0.877	0.871	0.875	0.859	0.906
	Rank	7	6	4	2	3	1	5
THA	Actual	0.596	0.589	0.521	0.516	0.525	0.498	0.548
	Relative	1.000	0.989	0.874	0.866	0.881	0.836	0.919
	Rank	7	6	3	2	4	1	5
UK	Actual	0.462	0.576	0.383	0.377	0.381	0.368	0.495
	Relative	0.801	1.000	0.664	0.653	0.661	0.639	0.859
	Rank	5	7	4	2	3	1	6
US	Actual	0.548	0.726	0.448	0.437	0.432	0.415	0.702
	Relative	0.756	1.000	0.617	0.603	0.595	0.571	0.967
	Rank	5	7	4	3	2	1	6

The mean absolute percentage error, (MAPE), actual figures must be multiplied by 10^{-2}.

b) Asymmetric Error Statistics

These conventional error statistics used in the previous subsection, ME, MAE, RMSE, and MAPE, are symmetric; ie., they give an equal weight to under-and-over-predictions of volatility of similar magnitude. However, many investors do not give equal importance to under and over prediction of volatility, Especially, in the pricing of the options, while underprediction of volatility is undesirable for a seller, overprediction of it is undesirable for a buyer. Following Pagan and Schwert (1990) and Brailsford and Faff (1996), to penalise under(over)-predictions more heavily, the following mean mixed error statistics are constructed:

$$MME(U) = \frac{1}{261}\left[\sum_{t=1}^{O}\left|\sigma_{f,w} - \sigma_{a,w}\right| + \sum_{t=1}^{U}\sqrt{\left|\sigma_{f,w} - \sigma_{a,w}\right|}\right] \tag{16}$$

$$MME(O) = \frac{1}{261}\left[\sum_{t=1}^{O}\sqrt{\left|\sigma_{f,w} - \sigma_{a,w}\right|} + \sum_{t=1}^{U}\left|\sigma_{f,w} - \sigma_{a,w}\right|\right] \tag{17}$$

where O is the number of over-predictions, and U is the number of under-predictions. MME(U) and MME(O) penalise the under-predictions and over-predictions more heavily, respectively.

It is expected that for an “unbiased” forecast model under and overprediction number should be equal to 50% of the time, ie. 50% of the time it underpredicts, and 50% of the time it overpredicts.

Results in Table 6 show that, except for RW, HM, and REG models, all of the models overpredict the volatility in all of the stock markets. However, among RW, HM, and REG, even though HM and REG overpredicts the volatility in 12 countries, RW over predicts the volatility only in 6 countries, ie. RW underpredicts in most of the countries. These results proved by MME conflict with the results provided by ME. The reason for the difference in the results can be explained as MME takes into account only the number of under and overpredictions, however for ME magnitude of the error is important. Thus for the difference it can be said that the models overpredict the volatility most of the time, but their forecast error are small.

If underprediction of the model is undesirable, HW is the best, and REG is the worst model. However, when the results for US is evaluated, it is observed that REG is an extreamly bad model for forecasting volatility of US market. If we drop the results for the US, and re-rank the models, REG is the best model, RW is the worst model, and HM is the third best model after WMA-12.

On the other hand, if overprediction is undesirable in the forecasting, ES model performs the best, and RW performs the worst. However, in this case, when we do not take into account the results for US, ranking does not change much, ranking of REG declines just from seven to six, and ES model is still the best model. According to MME(O), WMA-12 provides the third best performance after EWMA-12.

Table 6a. MMEU and MMEO

			RW	HM	MA-12	WMA-12	EWMA-12	ES	REG
BEL	MME(U)	Actual	2.269	1.682	1.949	1.922	2.024	2.003	1.705
		Relative	1.000	0.741	0.859	0.847	0.892	0.883	0.752
		Rank	7	1	4	3	6	5	2
	MME(O)	Actual	2.347	2.674	2.104	2.099	2.064	1.983	2.508
		Relative	0.878	1.000	0.787	0.785	0.772	0.741	0.938
		Rank	5	7	4	3	2	1	6
	%	Underestimaton	48.3	36.8	45.6	44.4	45.6	47.9	37.2
	%	Overestimation	51.7	63.2	54.4	55.6	54.4	52.1	62.8
CAN	MME(U)	Actual	2.308	1.958	1.953	1.915	2.029	1.935	1.919
		Relative	1.000	0.848	0.846	0.830	0.879	0.838	0.832
		Rank	7	5	4	1	6	3	2
	MME(O)	Actual	2.308	2.318	2.357	2.321	2.251	2.209	2.211
		Relative	0.979	0.984	1.000	0.985	0.955	0.937	0.938
		Rank	4	5	7	6	3	1	2
	%	Underestimaton	50.6	40.2	40.2	40.6	42.1	41.8	42.1
	%	Overestimation	49.4	59.8	59.8	59.4	57.9	58.2	57.9
DEN	MME(U)	Actual	2.280	1.632	1.893	1.874	1.920	1.869	1.592
		Relative	1.000	0.716	0.830	0.822	0.842	0.820	0.698
		Rank	7	2	5	4	6	3	1
	MME(O)	Actual	2.374	2.740	2.320	2.286	2.302	2.222	2.643
		Relative	0.867	1.000	0.847	0.834	0.840	0.811	0.965
		Rank	5	7	4	2	3	1	6
	%	Underestimaton	48.3	33.3	41.8	42.9	40.6	42.1	33.7
	%	Overestimation	51.7	66.7	58.2	57.1	59.4	57.9	66.3
FIN	MME(U)	Actual	3.453	4.291	2.724	2.693	2.783	2.676	3.695
		Relative	0.805	1.000	0.635	0.628	0.649	0.624	0.861
		Rank	5	7	3	2	4	1	6
	MME(O)	Actual	3.639	1.817	3.272	3.323	3.214	3.229	2.261
		Relative	1.000	0.499	0.899	0.913	0.883	0.887	0.621
		Rank	7	1	5	6	3	4	2
	%	Underestimaton	48.7	65.1	42.9	41.0	43.7	41.0	57.9
	%	Overestimation	51.3	34.9	57.1	59.0	56.3	59.0	42.1
GER	MME(U)	Actual	2.776	1.845	2.349	2.313	2.403	2.345	1.796
		Relative	1.000	0.665	0.846	0.833	0.866	0.845	0.647
		Rank	7	2	5	3	6	4	1
	MME(O)	Actual	2.806	4.004	2.585	2.560	2.482	2.406	3.626
		Relative	0.701	1.000	0.646	0.639	0.620	0.601	0.906
		Rank	5	7	4	3	2	1	6
	%	Underestimaton	49	27.2	45.2	45.2	44.8	46.4	29.5
	%	Overestimation	51	72.8	54.8	54.8	55.2	53.6	70.5
HON	MME(U)	Actual	3.792	3.769	3.192	3.132	3.241	3.303	3.446
		Relative	1.000	0.994	0.842	0.826	0.855	0.871	0.909
		Rank	7	6	2	1	3	4	5
	MME(O)	Actual	3.735	3.701	3.643	3.637	3.676	3.784	3.615
		Relative	0.987	0.978	0.963	0.961	0.972	1.000	0.955
		Rank	6	5	3	2	4	7	1
	%	Underestimaton	48.7	43.7	43.3	42.9	43.7	44.4	42.5
	%	Overestimation	51.3	56.3	56.7	57.1	56.3	55.6	57.5

Table 6b. MMEU and MMEO

			RW	HM	MA-12	WMA-12	EWMA-12	ES	REG
ITA	**MME(U)**	Actual	3.443	3.467	2.746	2.809	2.746	2.813	3.291
		Relative	0.993	1.000	0.792	0.810	0.792	0.811	0.949
		Rank	6	7	1	3	2	4	5
	MME(O)	Actual	3.358	2.419	3.154	3.190	3.082	3.020	2.600
		Relative	1.000	0.720	0.939	0.950	0.918	0.899	0.774
		Rank	7	1	5	6	4	3	2
	%	Underestimaton	51.3	56.3	44.8	45.6	44.8	45.2	53.6
	%	Overestimation	48.7	43.7	55.2	54.4	55.2	54.8	46.4
JAP	**MME(U)**	Actual	3.151	2.139	2.588	2.554	2.626	2.618	2.324
		Relative	1.000	0.679	0.821	0.811	0.834	0.831	0.737
		Rank	7	1	4	3	6	5	2
	MME(O)	Actual	2.982	3.890	3.024	2.960	3.009	2.808	3.254
		Relative	0.767	1.000	0.777	0.761	0.773	0.722	0.836
		Rank	3	7	5	2	4	1	6
	%	Underestimaton	51.7	30.7	41.4	42.1	41.8	45.6	38.7
	%	Overestimation	48.3	69.3	58.6	57.9	58.2	54.4	61.3
NET	**MME(U)**	Actual	2.446	2.383	2.357	2.288	2.431	2.342	2.191
		Relative	1.000	0.974	0.963	0.935	0.994	0.957	0.896
		Rank	7	5	4	2	6	3	1
	MME(O)	Actual	2.458	2.745	2.364	2.364	2.179	2.254	2.482
		Relative	0.896	1.000	0.861	0.861	0.794	0.821	0.904
		Rank	5	7	4	3	1	2	6
	%	Underestimaton	48.7	41.8	48.3	46.7	47.9	48.7	43.3
	%	Overestimation	51.3	58.2	51.7	53.3	52.1	51.3	56.7
PHI	**MME(U)**	Actual	3.560	2.412	3.059	2.999	3.093	3.030	2.363
		Relative	1.000	0.677	0.859	0.842	0.869	0.851	0.664
		Rank	7	2	5	3	6	4	1
	MME(O)	Actual	3.525	5.280	3.399	3.375	3.372	3.285	4.648
		Relative	0.668	1.000	0.644	0.639	0.639	0.622	0.880
		Rank	5	7	4	3	2	1	6
	%	Underestimaton	50.6	28.7	43.3	42.9	43.7	44.1	29.9
	%	Overestimation	49.4	71.3	56.7	57.1	56.3	55.9	70.1
SNG	**MME(U)**	Actual	2.977	2.298	2.488	2.426	2.527	2.477	2.296
		Relative	1.000	0.772	0.836	0.815	0.849	0.832	0.771
		Rank	7	2	5	3	6	4	1
	MME(O)	Actual	2.921	3.203	2.847	2.874	2.814	2.821	3.054
		Relative	0.912	1.000	0.889	0.897	0.879	0.881	0.954
		Rank	5	7	3	4	1	2	6
	%	Underestimaton	51.3	36.4	42.9	41.8	44.4	42.5	39.8
	%	Overestimation	48.7	63.6	57.1	58.2	55.6	57.5	60.2
THA	**MME(U)**	Actual	3.862	3.222	3.231	3.187	3.279	3.226	3.252
		Relative	1.000	0.834	0.837	0.825	0.849	0.835	0.842
		Rank	7	2	4	1	6	3	5
	MME(O)	Actual	3.741	4.268	3.824	3.828	3.808	3.675	4.004
		Relative	0.876	1.000	0.896	0.897	0.892	0.861	0.938
		Rank	2	7	4	5	3	1	6
	%	Underestimaton	51	37.2	43.3	41	43.7	42.5	41
	%	Overestimation	49	62.8	56.7	59	56.3	57.5	59

Table 6c. MMEU and MMEO

			RW	HM	MA-12	WMA-12	EWMA-12	ES	REG
UK	MME(U)	Actual	2.260	1.210	1.868	1.840	1.912	1.904	1.316
		Relative	1.000	0.535	0.826	0.814	0.846	0.843	0.583
		Rank	7	1	4	3	6	5	2
	MME(O)	Actual	2.177	3.387	2.088	2.040	2.083	1.956	2.967
		Relative	0.643	1.000	0.616	0.602	0.615	0.577	0.876
		Rank	5	7	4	2	3	1	6
	%	Underestimaton	52.9	26.1	44.8	44.8	44.4	47.1	31.0
	%	Overestimation	47.1	73.9	55.2	55.2	55.6	52.9	69.0
US	MME(U)	Actual	2.407	1.601	2.048	1.981	2.086	2.069	176.576
		Relative	0.014	0.009	0.012	0.011	0.012	0.012	1.000
		Rank	6	1	3	2	5	4	7
	MME(O)	Actual	2.406	3.585	2.163	2.170	2.119	2.015	11.222
		Relative	0.214	0.319	0.193	0.193	0.189	0.180	1.000
		Rank	5	6	3	4	2	1	7
	%	Underestimaton	50.2	28.7	46.4	44.1	46.4	46.7	34.5
	%	Overestimation	49.8	71.3	53.6	55.9	53.6	53.3	65.5

MME(U) and MME(O) are the mean mixed error statistics that penalise the underpredictions and overpredictions more heavily, respectively. *Actual* is the calculated error statistics. MME(U) and MME(O) actual figures must be multiplied by 10^{-2}. *Relative* is the ratio between the actual error statistic of a model and that of the worst performing model. The best performing model has a rank 1.

IV. CONCLUSION

The present paper employs seven different models, a random walk model, a historical mean model, moving average models, weighted moving average models, exponentially weighted moving average models, an exponential smoothing model, and a regression model, to forecast out-of-sample within-week standard deviations in fourteen countries.

In the evaluation of the performances of the models, both symmetric and asymmetric statistics are used. The results show that symmetric statistics, the mean absolute error, the root mean squared error, and the mean absolute percentage error, consistently favour ES model. However, symmetric statistics show that HM and RW models perform poorly.

On the other hand, even though according to asymmetric statistics RW is consistently the worst model, asymmetric statistics do not prove consistent results for the best performing model.

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