MULTIVARIATE QUALITY CONTROL: A HISTORICAL PERSPECTIVE

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ABSTRACT

One of the most powerful tools in the quality control is the statistical control chart. First developed in the 1920's by Walter Shewhart, the control chart found widespread use during World War II and has been employed, with various modifications, ever since.

There are many processes in which the simultaneous monitoring or control of two or more quality characteristics is necessary. Process monitoring problems in which several variables are of interest are called "multivariate quality control (or process monitoring)" problems. Some of the problems areas in the use of multivariate statistical techniques for process control are multivariate analogues of univariate areas. The first original study in multivariate quality control was introduced by Hotelling (1947). Three of the most popular multivariate control statistics are Hotelling's T^2, the MEWMA (Multivariate Exponentially-Weighted Moving Average) and the MCUSUM (Multivariate Cumulative Sum).

This study covers both the motivation for multivariate quality control and a discussion of some of the techniques currently available. The emphasis focuses primarily on the developments occuring since the mid-1980's. The topics covered by this paper are control charts, especially the use of Hotelling's T^2 control chart to monitor the mean and charts for the process variability for Phase I and Phase II of a process. Also, new practical approaches (Mason et. al., 1997; Mason and Young, 1999) for interpreting multivariate T^2 control chart signals were developed recently will be discussed.

Introduction

Statisticmal quality control (SQC) and improvement is a branch of industrial statistics which includes primarily, the areas of acceptance sampling, statistical process monitoring and control (SPC), design of experiments, and capability analysis. We have especially worked on SPC. The modern origins of statistical process control (SPC) begin with a letter Walter

Shewart wrote in 1924. In 1926 Shewart published the paper “Quality Control Charts” in the *Bell System Technical Journal*, and the word “control” entered the lexicon of the quality engineer. The full exposition of Shewhart’s ideas appears in his book *Economic Control of Manufactured Product* and is reinforced in a second book, *Statistical Method from the Viewpoint of Quality Control*, which authored with W. Edwards Deming (Hunter, 1999).

Considerable enthusiasm was generated on the subject of statistical quality control during and after World War II. Much of this was the result of requirements of both the British and American goverments for the use of quality control with regard to government contracts during the war and many concerns, having been exposed to these techniques, found them equality applicable with resumption of non-military production after the war.

The statistical control chart is one of the most powerful tools in the quality control. First developed in the 1920’s by Walter Shewhart, the control chart found widespread use during World War II and has been employed, with various modifications ever since (Jackson,1985). A univariate chart selection guidelines is given in Figure 1.

Figure 1

Multivariate Methods, Effect of Estimation Error,Short-Run Method, Autocorrelated Data, Variable Sampling Methods, Economic Design Methods, Change-Point Estimation, Engineering Process Control and SPC, Nonparametric Approaches are some of the most active research areas in SPC. We are mainly interested on Multivariate Methods in this study.

There are multiple variables for defining the quality of the process. The rapid growth of data – analysis technology and the use of online computers for process monitoring have led to an increased interest in the simultaneous discussion of several related quality characteristic or process variables. These tecniques are often called to as multivariate statistical process control procedures.

Many issues need to be considered when determining whether to use multivariate process control techniques to monitor an industrial process. Some of the problem areas in the use of multivariate statistical techniques for process control are multivariate analogus of univariate areas.

Some process steps are necessary for evaluating a process correctly. Seven steps are proposed by Juran (1988) which is given in Figure 2.

Figure 2

2. Multivariate Quality Control

The first original study in multivariate quality control was introduced by Hotelling (1947) (Hotelling, 1947). He applied his procedures to bombsight data during World War II. Before this study, he wrote a paper on T^2- test procedures for multivariate population in the 1931 (Hotelling, 1931).

Subsequent papers dealing with control procedures for several related variables include Montgomery (1996), Hotelling (1947), Hicks (1955), Jackson (1956), (1957), (1959), (1985), Hawkins (1981), Alt (1982), Alt (1984), Alt (1985), Jackson (1985), Healy (1987), Murpy (1987), Crosier (1988), Pignatiello and Ranger (1990), Doganaksoy, Tucker, (1991), Hawkins(1991), Chua, Montgomery (1992), Lowry, Woodall, Champ, Rigdon, (1992), Tracy, Young and Mason (1992), Hawkins (1993), Hayter, Tsui, (1994), Lowry and Montgomery (1995), Liu (1995), Mason, Tracy and Young, (1995), Sullivan, Woodall, (1996), Mason et. al. (1997), Mason, Tracy and Young (1997), Mason and Young (1999).

Unfortunately, the momentum was not enough to carry through the 1960's, when quality seemed to take a back seat to other objectives. The use of multivariate quality control techniques was hampered at that time the lack of computational tools (computer, software etc.)

Three of the most popular multivariate quality control statistics are Hotelling's T^2, the MEWMA (Multivariate exponentially-weighted moving average), MCUSUM (Multivariate Cumulative Sum).This study covers both the motivation for multivariate quality control and a discussion of some of the techniques currently available. The emphasis focuses primarily on the developments occuring since the mid-1980's. The topics covered by this paper are control charts, especially the use of Hotelling's T^2-control chart to monitor the mean and charts for process variability for Phase I and Phase II of a process, Also new practical approaches for interpreting multivariate T^2- control chart signals were developed recently will be discussed (Mason et. al., 1997).

Various univariate charts, like median charts, midrange, CUSUM, MA, EWMA,... etc. have been developed and used in many industries. Using these charts provide only partial information about the process. This is especially true when the quality characteristics are highly correlated.

Suppose that, a process has two quality characteristics (x_1,x_2) that together determine the usefulness of the part. As both quality characteristics are measurements, they could be monitored by applying the usual $\bar{x}$ chart to each characteristic. The process is considered to be in control only if the sample means $\bar{x}_1$ and $\bar{x}_2$ fall within their respective control limits. Monitoring these two quality characteristics independently can be very misleading (Montgomery, 1996). The probability that either $\bar{x}_1$ or $\bar{x}_2$ exceeds 3 sigma control limits is 0.0027 (α). However the probability of both charts simultaneasly in control is not (1-α). If a process is in-control, the probability of p means ploting in control is $(1-\alpha)^p$, Thus the joint probability of a type 1 error is much larger $(1-\alpha)^p$

3. Multivariate Quality Control Charts

Multivariate charts like Hotelling's T^2, MEWMA , MCUSUM, take this correlation into account in monitoring the mean vector or variance-covariance matrix. Multivariate charts are less popular than univariate control charts. Besides the relative difficulty involved in their computation, there are some additional difficulties which diminish their appeal:

- Unlike the univarite case, the scale of the values displayed on the multivariate chart is not related to the scales of any of the monitored variables
- Once an out-of-control signal is given by a multivariate chart, it may be difficult to determine which variable caused this signal. Various approaches to this problem have been proposed. (Woodall and Montgomery, 1999). More complicated operations are required to determine the origin of the signals.

3.1. Shewhart Multivariate Control Charts

The first step in constructing a Shewhart Multivariate Chart requires analysis of a preliminary set of data that is assumed to be in a state of statistical control. These analysis are Phase I and Phase II analysis. The distinction between these two phases of data collection is important. The first phase of data collection should utilize a very large sample of data so that parameters and control limits are well estimated for Phase II. Phase I charts are for defining what is meant by in control and charts that are used in the second phase are for monitoring the process (Mason et. al., 1997).

3.1.1. χ^2 Charts

If the true parameters of a probability distribution are known, the χ^2 distribution may be used. Suppose that, x_1 and x_2 are the variables from a normal distributed process and μ_1 and μ_2 are the population means of these variables. x_1 and x_2 are the sample means, under the assumption of knowing the standart deviations ($\sigma_{1\sigma 2}$) and the covariance (σ_{12}).

$$\chi_0^2 = \frac{n}{\delta_1^2 \cdot \delta_2^2 - \delta_{12}} \left[\delta_2^2 (x_1 - \mu_1) + \delta_1^2 (x_2 - \mu_2) - 2\delta_{12}(x_1 - \mu_1)(x_2 - \mu_2)\right]$$

The χ_o^2- statistic follows a χ_o^2- distribution with 2 freedom- degrees. Monitoring the process and detecting the out-of-control points depends on constructing correct control limits. The upper control limit (UCL) for the χ^2 Charts is given by:

$$\text{UCL} = \chi_{o\,\alpha,2}^2$$

χ_o^2- statistic is calculated for each sample. We can define a χ_o^2matrix as follows:

$$\chi_o^2 = n\,(x - \mu)^l\, \Sigma^{-1}\,(x - \mu)$$

When the true population values are not known, Hotelling's T^2 control chart is used instead of χ_o^2 charts. Generally, Σ and μ are predicted from a sample. When p is the number of variables, the covariance matrix (p*p) is as follows:

$$S = \begin{bmatrix} S_1^2 & S_{12} & S_{13} & \cdots & S_{1p} \\ & S_2^2 & S_{23} & \cdots & S_{2p} \\ & & S_3^2 & \cdots & \cdots \\ & & & & S_p^2 \end{bmatrix}$$

If the process is in control, S will be the unbised prediction of Σ .

3.1.2 Hotelling's T^2 Charts

In multivariate SPC, a signal can be caused by variety of situations. For example, an observation on one of the p variables may be out of control. Similarly, the signal may be due to a relationship between two or more of the variables. Worse yet, a signal maybe produced by combinations of these two situations, with some variables being out of control and others have counter-relationships (Mason et. al., 1997).

Hotelling, in his paper "Multivariate Quality Control" was the first to consider the problem of analyzing a correlated set of variables in his analysis. He developed a control procedure based on a concept referred to as statistical distance, a generalization of the T statistic. The statistic was later named Hotelling's T^2 in his honor. About the same time, Mahalanaobis developed a similar statistical distance that is reffered to as Mahalanobis' distance. Between these two distances exist a constant. T^2 has emerged as an extremely useful metric for multivariate process control. T^2 statistic can be constructed in most software packages such as Qual Stat.

Consider a process that generates an uncorrelated bivariate observation (x_1 and x_2). The formula for the standadized values for each variables:

$$\frac{(x_1-\mu_1)^2}{\delta_1^2} \text{ and } \frac{(x_2-\mu_2)^2}{\delta_2^2}$$

and all points satisfying the relationship:

$$\frac{(x_1-\mu_1)^2}{\delta_1^2}+\frac{(x_2-\mu_2)^2}{\delta_2^2}=SD^2$$

This statistical measures is known as statistical distance. The graph of this equation is an ellipse. Any point inside the ellipse will have a statistical distance less than SD. Likewise any point located outside the ellipse will have a statisitcal distance greater than SD. (Figure 3a,3b)

Figure 3a: A Scatter Plot of Multivariate Data Composed of Two Variables

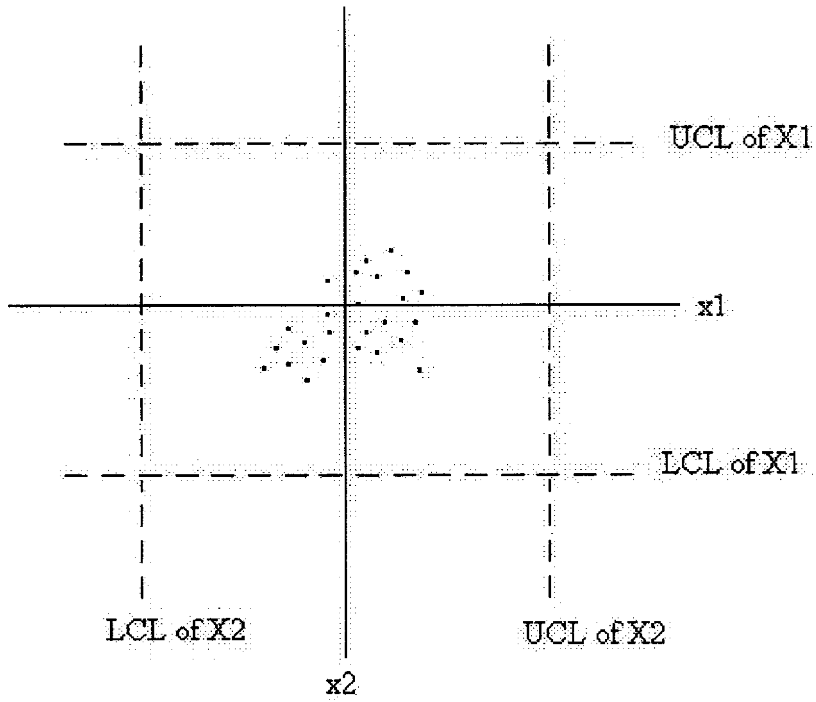


Figure 3b

One assumption fundamental to using Hotelling's T^2 to describe the behavior of statistical distance is the observation vector must follow a multivariate normal distribution. Under this assumption (x_1,x_2) can be described jointly as a bivariate normal.

Using matrix notation:

$$(x-\mu)' \Sigma^{-1} (x-\mu) = SD^2$$

Where $x' = (x_1,x_2)$ $\mu' = (\mu_1,\mu_2)$ and Σ^{-1} is the inverse of the covariance matrix. This formula is the quadratic form of vector $(x-\mu)$ that represents the statistical distance. There for this equation is known as Hotelling's T^2 statistics:

$$T^2 = n(\bar{x} - \mu)' S^{-1} (\bar{x} - \mu) = SD^2$$

Hotelling's T^2 is a well-known control statistic for monitoring and control a multivariate process. This statistic has some characteristics. Among its characterisitcs is its dependency on the correlations among the process variables. When no correlation is present there is no need to construct Shewart charts monitoring with a T^2 statistic is quite enough. If correlation does exist, T^2 can be decomposed into orthogonal components. These components show how each variable is associated with the remaining variables (Mason et. al., 1997). Having knowledge of these relationships is very helpful in interpreting multivariate control chart - signals.

3.1.3. Multivariate Cumulative Sum (CUSUM) and Multivariate Exponentially-Weighted Moving Average (MEWMA) Charts

Shewart–type control charts (the chi-square and Hotelling T^2 charts) use information from only the current sample, so consequently, they are relatively insensitive to small and moderate shifts in the mean vector. Cumulative sum and EWMA control charts were developed to provide more sensitivity to small shifts in the univariate case, and they can be extended to multivariate quality control problems.

Cumulative Sum control charts are widely used in industry because they are powerful and easy to use. They cumulate recent process data to quickly detect out-of-control situations. CUSUM procedures will usually give tighter process control than classical quality control charts. A CUSUM signal does not mean that the process is producting bad product. Rather it means that action should be taken so that the process does not produce bad product. CUSUM procedures give an early indication of process change, they are consistant with a management philosophy that encourages doing it right the first time (Lucas, 1985).

Besides CUSUM charts, some multivariate CUSUM charts are developed, which are more sensitive to out-of-control signals. Some MCUSUM charts are as given;

- COT is the CUSUM of T chart proposed by Crosier (1988)
- CCU is Crosier's „multivariate CUSUM“
- MCX is Woodall and Ncube's (1985) MCUSUM applied to X

- MCZ is the MCUSUM proposal applied to Z (Z is the residual when variable (i) is regressed on all other variable)
- ZNO İS THE Quclidean norm of Z CUSUM's (Hawkins, 1991)

Lowry (1992) has developed the multivariate chart of EWMA (MEWMA). The equation is for the MEWMA are;

$$Z_i = \lambda_i x_i + (1-\lambda) Z_{i-1} \quad 0 < \lambda \leq 1 \text{ ve } Z_o = 0$$

$$T_i^2 = Z_i^1 \Sigma_{zi}^{-1} Z_i \, \Sigma_{zi} = \frac{\lambda}{2-\lambda} [\, 1 - (1-\lambda)^{2i} \,] \, \Sigma$$

Where T_i^2 is the plotted statistic, Σ_{zi} is the covariance matrix and λ is the EWMA weighted matrix. The MEWMA allows for the user to define specific weights for each variable being measured using the λ matrix. Selecting an approprite values of λ to use is not an easy decesion.

MEWMA and MCUSUM charts tend to have inertia that later data points carry with them. As a result, when a trend occurs on one direction of the target mean and a resulting shift occurs in the other direction of the target mean, the two types of charts will not pick up the shift immediately.

4. Detection and Interpretation Techniques for Out-of-Control Signals in Multivariate SPC

One difficulty encountered with any multivariate control chart is practical interpretation of an out-of-control signal. The complexity of multivariate control charts and the cross-correlation among variables make it difficult for analysis of assignable causes to the out-of-control signal.

Several techniques have been developed that assist in the interpretation of out-of-control signals.

4.1. Principle Component Analysis

The significance of the correlation between variables(the correlation coefficient), defines the linear association between the two variables. However, the correlation coefficient does not provide a measure of the relationship among a group of variables. To define this multivariate relationship, a tecnique more commonly referred to as principle component analysis is used. Therefore, the principle component analysis can be thought of as a multivariate correlation analysis (PCA). A detailed description of the method of principal components and its relationship to quality control may be found in Jackson's papers (Jackson, 1985).

The goal of principal component analysis is to simplify the complexity of the data. If a large number of factors are needed to define the dimensionality of the data, then there is very little need for principal component analysis. The purpose of the principal component analysis is to reduce the overall dimensionality in the data.

The principal components of factors as they are often referred are set of linear combinations of the variables that redefines the existing variable space. The first principle component is the combination of variables that explains the greatest amount of variation. The second principal component defines the next largest amount of variation and is independent to the first principal component. Therefore, the two principal components are independent. Principal components are independent to all other principal components. There can be as many possible principal components as there are variables.

Any principal component analysis is made up of the following three distinctive steps:

- Calculation of the correlation matrix. The correlation matrix should be reviewed to identify possible subgroupings of variables that are correlated. This step can also be used to eleminate variables that are correlated to any of the other variables.
- The next step is to calculate the principal components. At this point, you will be able to determine the dimensionality of the data set.
- The last step of the process is calculating the transformed data set. During this step, you will be able to view the data in a 3D space to determine potential multivariate relationships in the data.

The principal components do not provide an overall measure of depature of the multivariate data from the norms. It can be difficult to assign a meaningful interpretation to these principal components. Therefore princaple component charts may complement but not replace the multivariate T^2 - charts (www.incontroltech.com).

4.2. The Step-Down Procedures

The new observation Y, has a distribution with mean $\mu_y = [\mu_{y1}, \mu_{y2.....}\mu_{yq]}$. Its covariance matrix Σ is assumed to be as in the reference population. We mentioned that the step-down procedure assumes an a priori ordering among the subsets of variables.The mean vector μ_x of the reference population is partitioned similarly into μ_{x1}, $\mu_{x2,...}\mu_{xq}$. p_j is the number of variables in the j^{th} subset. q_j is the number of variables in μ_{yi} and μ_{xj}. The step-down procedure tests sequentially;

$$H_0^{(1)} : \mu_{y1} = \mu_{x1} \text{ versus } H_A: \mu_{y1 \neq \mu x1}$$

$$H_0^{(2)} : \mu_{y2} = \mu_{x2} \text{ versus } H_A: \mu_{y2 \neq \mu x2}$$

$$\cdots$$

$$\cdots$$

$$\cdots$$

$$\cdots$$

$$H_0^{(q)} : \mu_{yq} = \mu_{xq} \text{ versus } H_A: \mu_{yq \neq \mu xq}$$

The test calls for the computation of a series of statistics T_j^2 defined as

$$T_j^2 = (\bar{Y} - \bar{X})^1{}_{(j)} S^{-1}{}_{(j)} (Y-X)_{(j)}, j=1, 2, 3,.......q$$

For the test for j^{th} hypothesis we compute the statistic

$$G_j^2 = \frac{T_j - T_{j-1}}{\left(1 + T_{j-1}^2\right)/\left(n_2 - 1\right)} \quad j=1, 2, 3, ...q \text{ Where } T^2_0=0$$

By computing a upper control limit (UCL), the hypothesis can be tested. If at least one of the G_j values exceeds the corresponding UCL_j, the process is declared statistically out-of control (Fuchs and Kennet, 1998).

4.3. Graphical Techniques

Discrimination analysis procedures allows for the removel of in-control variables from out-of-control variables in order to determine where assignable causes of variation are occuring. Univariate signaling approaches to interpretation have been even more effective. This is particularly useful with the T^2 control chart. This procedures work by partitioning the multivariate control chart into the contributions of each variable (www.sys.virginia.edu/mqc).

Several graphical methods for coping with these procedures have recently been proposed. Two methods are presented in this section:

- The starplots
- Multivariate Profile Charts

4.3.1. Starplots

The starplot was first developed at the SCS corporation as an enhancement to the Multivariate Control Charts available through the STATGRAPHICS (3.0, 1988) statistical software package. The starplots consist of stars positioned vertically at the group T^2- value (Fuchs and Kennet, 1998).

Each multivariate observation is portrayed in the starplot by a star with the rays proportional to the deviations of the individual variables from the minimal values across all observations (or groups of observations). Starplots give no indication of the statistical significance of the ray's length. An example is given in Figure 4.

Figure 4

4.3.2. Multivariate Profile (MP) Charts

Fuchs and Benjamini (1991) review a number of methods for displaying simultaneously a multivariate chart and several univariate charts. They also present an alternative method, termed a multivariate profile chart (Mason et. al., 1997).

The MP–charts were proposed as an improvement over the starplot and other methods proposed in the literature (Fuchs and Kennet, 1998). The concept of this chart is that instead of just a dot on the T^2 control chart, a small bar chart is plotted at he height of the value of T^2. The bar chart contains the values of several univariate statistics.

4.4. MYT Decomposition

The major drawback of most multivariate control chart procedures is that they do not directly provide the information an operator needs when the control charts signals.To be more specific, the operator needs to know which variable(s) caused the out-of-control signal (Mason et. al., 1997).

Many authors have suggested using decomposition techniques. These techniques are used mostly on T^2 statistic. Murpy (1987) divided p variables into two subsets and the statistic that he used is given by;

$$D_J = T^2 - T_J^2$$

T_J^2 is the T^2 for the J variables in the suspected subset. D_J, follows a F-distribution. A disadvantage of this method is that the number of possible selections is large than the number of variables (p).

The last method, which is developed by Mason, Tracy and Young (1995), decomposes the T^2 statistic into orthogonal components when correlation does exist in data. This orthogonal decomposition, termed the MYT decomposition. MYT, encompasses many of the research finding on the interpretation of T^2 signals. Monitoring of the MYT decomposition terms, can prove to be very beneficial in enhancing the sensitivity of the T^2 statistic to detect such shifts. The success of the MYT is because of using the fact that the decomposition terms are functions of the residuals of the estimated regression models, constructed from all possible subsets of the process variables (Mason et.al., 1997). These residuals can improve the sensitivity of both abrupt process changes and graduals process shifts. The sensitivity depends on describing the relationships between process variables correctly, describing a good model, depends on selecting correct process variables. Choices of the variables often are made by process engineers, who use theoretical knowledge or expert opinion of the process. Deciding the functional forms of the variables are also very important. (logarithmic form, inverse function)

MYT decomposition includes orthogonal components. These components consist of a series of conditional and unconditional T^2 terms. The general form of the j^{th} unconditional T^2 is given by;

$$T^2_J = \frac{(\bar{x}_J - x_J)^2}{s_J^{\,2}}$$

x_J, is the j^{th} component of the signaling observation vector and x_J and s_J^2 are its corresponding mean and variance estimated as determined from the historical data set. Each unconditional term is itself a T^2 statistic and thus its value can be compared to an F distrıbution.

$$T_j^2 \sim \frac{n+1}{n} F_{(1,\, n-1)}$$

Similarly the general form of the conditional T^2 term is given by;

$$T_{J.\,1,\,2,\,\ldots\ldots,J-1}^2 = \frac{\left(x_j - \bar{x}_{J.1,2,\ldots J-1}\right)^2}{s^2_{J.1,2,\ldots j-1}}$$

Also each conditional term itself is a T^2 statistic and its value can be compared to an F distribution.

$$T_{J\,.\,1,\,2,\,\ldots\ldots,J-1}^2 \sim \frac{(n+1)(n-1)}{n(n-k-1)} F_{(1,\,n-k-1)}$$

k is the number of conditioned variables. The conditional T^2 term has the form of a squared standardized residual (residual /standart deviation). The conditional T^2 also can be expressed as;

$$T_{J\,.\,1,\,2,\,\ldots\ldots,j-1}^2 = \frac{\left(\hat{T}_j - T_j\right)^2}{1 - R^2_{J.1,2,\ldots j-1}}$$

Where T_j is the unconditional term, $\hat{T}_j$ is the corresponding predicted values of T_J obtained using the regression model, $R^2_{J.1,2..J-1}$ is the square of the multiple correlation coefficient between x_j and $x_1, x_2, \ldots, x_{J-1}$.

Sometimes large residuals $\left(\hat{T}_j - T_j\right)$ may not be truely significant and it is possible to accept an out-of control signal as being in control. This result would be due to the large error of the regression fit rather than due to a process shift (Mason and Young, 1999).

5. Conclusion

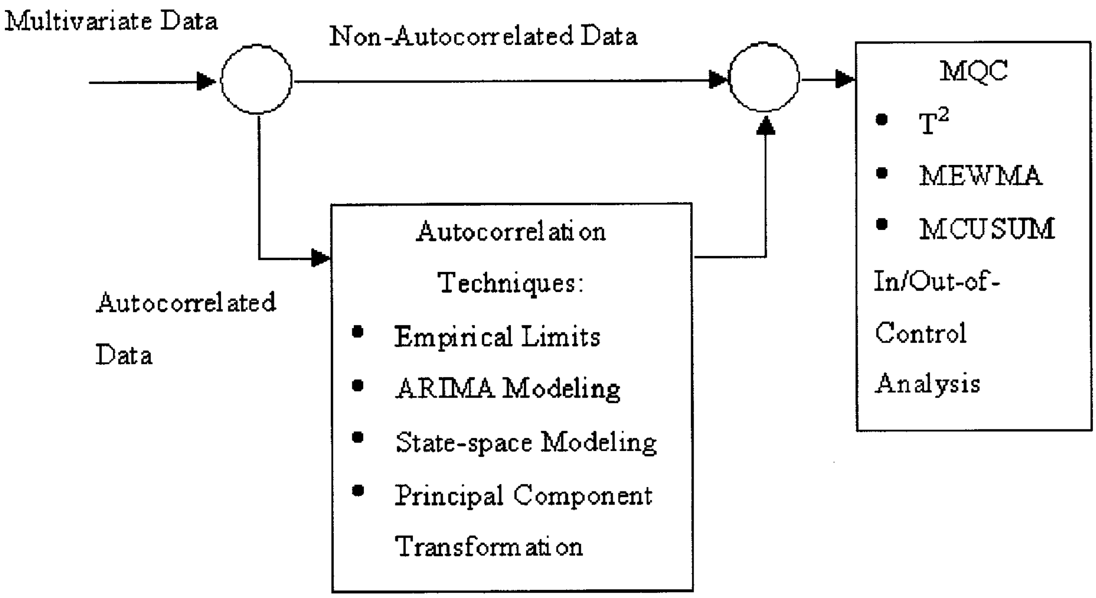
We have discussed, the development and use of multivariate quality control tools. This study has reviewed the state of the art in control charts for multivariate observations and some aspects of multivariate process monitoring. This is a large and diversified area of research and application and as measuring and sensoring capabilities develop, interest in multivariate quality control will continue to grow.

We conclude with a highlight of main points presented in this study, that are specific to multivariate quality control:

1. Hotelling's T^2 is a important chart for multivariate quality control. It can be decomposed into an overall measure of distance of the group means from the target T^2 and internal measure of variability.
2. The other of the most popular multivariate control statistics are the MEWMA and the MCUSUM.
3. The Hotelling multivariate control-chart procedure is based on only the most recent observation, therefore it is insensitive to small and moderate shifts in the mean vector. But several multivariate CUSUM procedures and one multivariate EWMA procedure use additional information from recent history of the process.
4. The analysis of T^2- charts needs to be complemented with additional graphical displays. (Starplot and MP charts)

5. The detection and interpretation of the variables responsible for a multivariate out-of-control. Signal can be realized either by graphical univariate methods or by multivariate decomposition of the T^2.
6. Principal components can be useful for multivariate quality control, especially when they lead to new dimensions that can be naturally interpreted.
7. MYT is the last decomposition technic which is developed by (Mason et. al., 1995). MYT decomposes the T^2 –statistic to its conditional and unconditional terms. These terms can decide the variable(s) which is out-of control and improve the sensitivity of T^2-statistic.
8. Graphical techniques avaible through computer technology and software packages provide efficient and effective environments for multivariate quality control.
9. Many softwares are avaible for computing multivariate statistics and drawing charts. Minitab, SAS, Statgraphics and Qual Stat are some of them. Qualstat is the first software package to provide a comprehensive solution to Multivariate Statistical Process Control. Qual Stat all the necessary tools to perform Multivariate SPC including Hotelling T^2 charts, principal component charts and the decomposition of a signal into resulting variables. Qual Stat is loaded with many other features to assist with your process evaluation.
10. In real World applications of Multivariate Quality Control and univariate SPC, the models presented are not entirely accurate. They do give a good estimation of the process or system, but they make so many assumptions about the data. Data sets are mostly autocorrelated. Therefore in order to monitor autocorrelated data, without a series of false alarms, a better tool is required than the general MQC control charts. (Figure 5)

Figure 5: Multivariate Control Chart Selection Guidelines



Is the data correlated?

No Yes

Variables or Attributes?

Variables (Measurable) Attributes (Countable)

Sample size Data Type

n>1 n=1 Fraction (Binomial Data Defects (Poisson Data)

Sample Number Sample Number Sample Number Sample Number

Large Small Large Small Large Small Large Small

$\bar{x}$, R CUSUM X CUSUM p CUSUM c CUSUM

$\bar{x}$, S EWMA MR EWMA np EWMA u EWMA

p p, u

BIBLIOGRAPHY

[1] Fuchs, C., and Kennet, R., (1998), **Multivariate Quality Control, Theory and Applications**, First Edition, U.S.A, Marcel Dekker, Inc., p.85

[2] Hawkins, D. M., (1991), "Multivariate Quality Control Based On Regression – Adjusted Variables", **Technometrics**, Vol.33 Issue1, pp 61-76

[3] Hawkins, D.M., (1993), "Regression Adjustment for Variables in Multivariate Quality Control", **Journal of Quality Technology**, Vol.25, No.3

[4] Hotelling, H., (1931), "The Generalization of Student's Ratio", **Annals of Mathematical Statistics**, n.2, pp.360-378

[5] Hotelling, H., (1947), "Multivariate Quality Control Illustrated by Air Testing of Sample Bombsights", C.Eisenhart et. al. Pp.111-184

[6] Hunter, J. S., (Dec 1999), "Statistical Process Control", **Quality Progress**,p.54

[7] Jackson, E. J., (1985), "Multivariate Quality Control", **Communication Statistics – Theory and Methods**, Vol.14 Issue 11, pp 2657 – 2688

[8] Liu, R.Y. (1995), "Control Charts for Multivariate Processes", **JASA,** Vol.90, No.432, pp. 1380-1387.

[9] Lowry, C.A.; Woodall, W. H. ;Champ, C.W. and Rigdon, S.E. , (1992), "A Multivariate Exponentially Weighted Moving Average Control Chart", **Technometrics,** Vol.34, No.1, pp 46-53

[10] Lucas, J., (1985), "Cumulative Sum (CUSUM) Control Schemes", **Commun. Statist.-Theor. Meth.**, 14, (11), pp.2689-2704

[11] Mason, R.L., (1999), "Book Reviews:Multivariate Quality Control: Theory and Applications", **Technometrics**, Vol.41, Issue 2, pp 168-170

[12] Mason, R.L.; Champ, C.W., Tracy, N.D.; Wierda, S.J. and Young, J.C., (1997), "Assesment of Multivariate Process Control Techniques", **Journal of Quality Technology**, Vol.29, No.2, pp 140-143

[13] Mason, R.L., Tracy, N.D. and Young, J.C., (1997), "A Practical Approach for Interpreting Multivariate T^2 Control Chart", **Journal of Quality Technology**, Vol.29, No.4, pp396-406

[14] Mason, R. L.; Young, J. C., (1999), "Improving the Sensitivity of the T^2 Statistic in Multivariate Process Control", **Journal of Quality Technology**, Vol.31, Issue 2, pp 155-165

[15] Montgomery, D. C., (1996), **Introduction to Statistical Quality Control**,Third Edition, John Wiley & Sons

[16] Montgomery, D. C., Woodall, W. H., (Oct 1999), "Research Issues and and Ideas in Statistical Process Control", **Journal of Quality Technology**, Vol.31, No:4

[17] Morrison, D. F., (1976), **Multivariate Statistical Methods**, second edition, McGraw-Hill, pp 128-136

[18] Pignatiello, J. J. and Runger, G. C., (1990), "Comparisons of Multivariate CUSUM Charts", **Journal of Quality Technology**, Vol.22, No.3, pp 173-186. www.sys.virginia.edu/mqc, www.incontroltech.com.